**Chapter 08**

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## 8.1 Integration by Parts

Alternatively, this can be written as

The order of precedence of is given by L.I.A.T.E., which stands for Logarithms, Inverse, Algebraic, Trigonometric, Exponential. However, if both and are algebraic functions, the function with the higher integer power is to be considered as .

Using, ,

We know,

Evaluate:

i)

Let

ii)

Let

iii)

Let

iv)

Let .

Example 5:

Obtain the reduction formula for:

1.

This can also be written as:

where

Let

or

2.

This can also be written as:

where

Let

or

This is the same as the result found from the previous method.

Exercise 8.1

1 – 50

5.

Let .

In general,

29.

Let .

Let

46.

Let .

Summary:

## 8.2 Trigonometric Integrals

where and are integers

Case 1: If is odd ( is even), use the identity

Case 2: If is odd ( is even), use the identity

Case 3: If both and are even, use the identities and

.

Case 4: If both and are odd, use one of the identities or .

Exercise 8.2

1 – 22

11.

Let .

20.

Let

Let

Exercise 8.2

23 – 32

23.

27.

Let .

## 8.3 Trigonometric Substitution

:

:

:

:

:

Exercise 8.3

1 – 28

5.

Let

9.

Let .

Exercise 8.3

15 – 34

17.

Let .

Exercise 8.3

35 – 48

Use any appropriate substitution method, and then trigonometric substitution.

35.

Let

Let

44.

Let

Let

## 8.4 Integration of Rational Functions by Partial Fractions

A rational function is a function that can be expressed as a rational fraction.

A proper rational function is one in which the degree of the numerator is less than that of the denominator.

An improper rational function is one in which the degree of the numerator is greater than or equal to that of the denominator.

For proper rational functions,

a) If the denominator is a linear factor (such as ),

b) If the denominator is a repeated linear factor (such as ),

c) If the denominator is a quadratic factor (such as ,

d) If the denominator is a repeated quadratic factor (such as ),

For improper rational functions,

e)

### General Method

Example 1:

Equating the like terms on both sides,

- (i)

- (ii)

Solving equations (i) and (ii),

Exercise 3:

Equating the like terms on both sides,

Let - (i)

Substituting equation (i) here,

Exercise 22

Equating the like terms on both sides,

### Heaviside Coverup Method

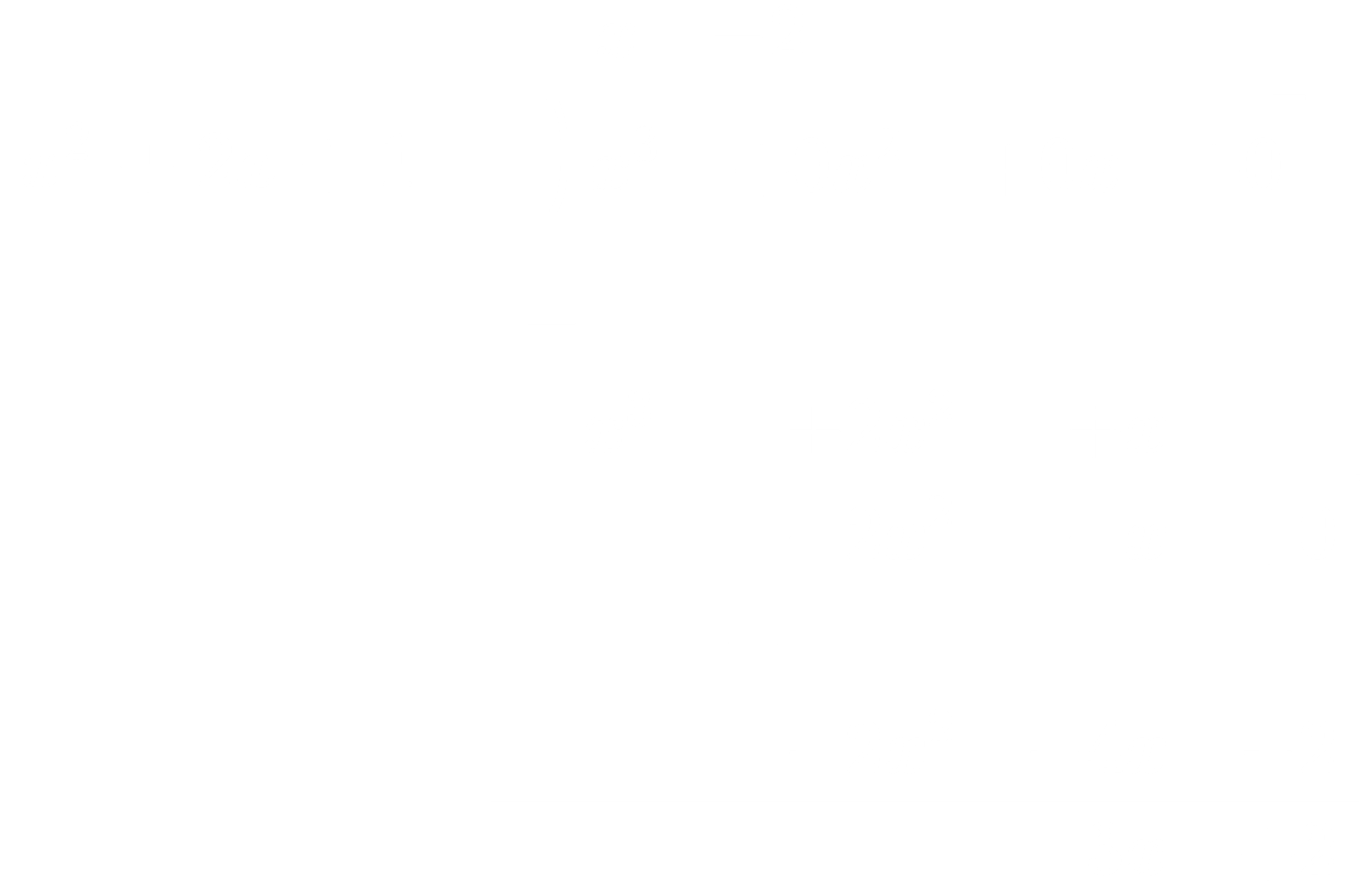
This method is only applicable for linear factors and repeated linear factors for proper functions.

Example 6

Let .

Let .

Exercise 17



Using the Heaviside Cover-Up Method,

Let .

Let .

Example 8

- (i)

Using the Heaviside Cover-Up Method,

Let .

Differentiating equation (i) on both sides,

- (ii)

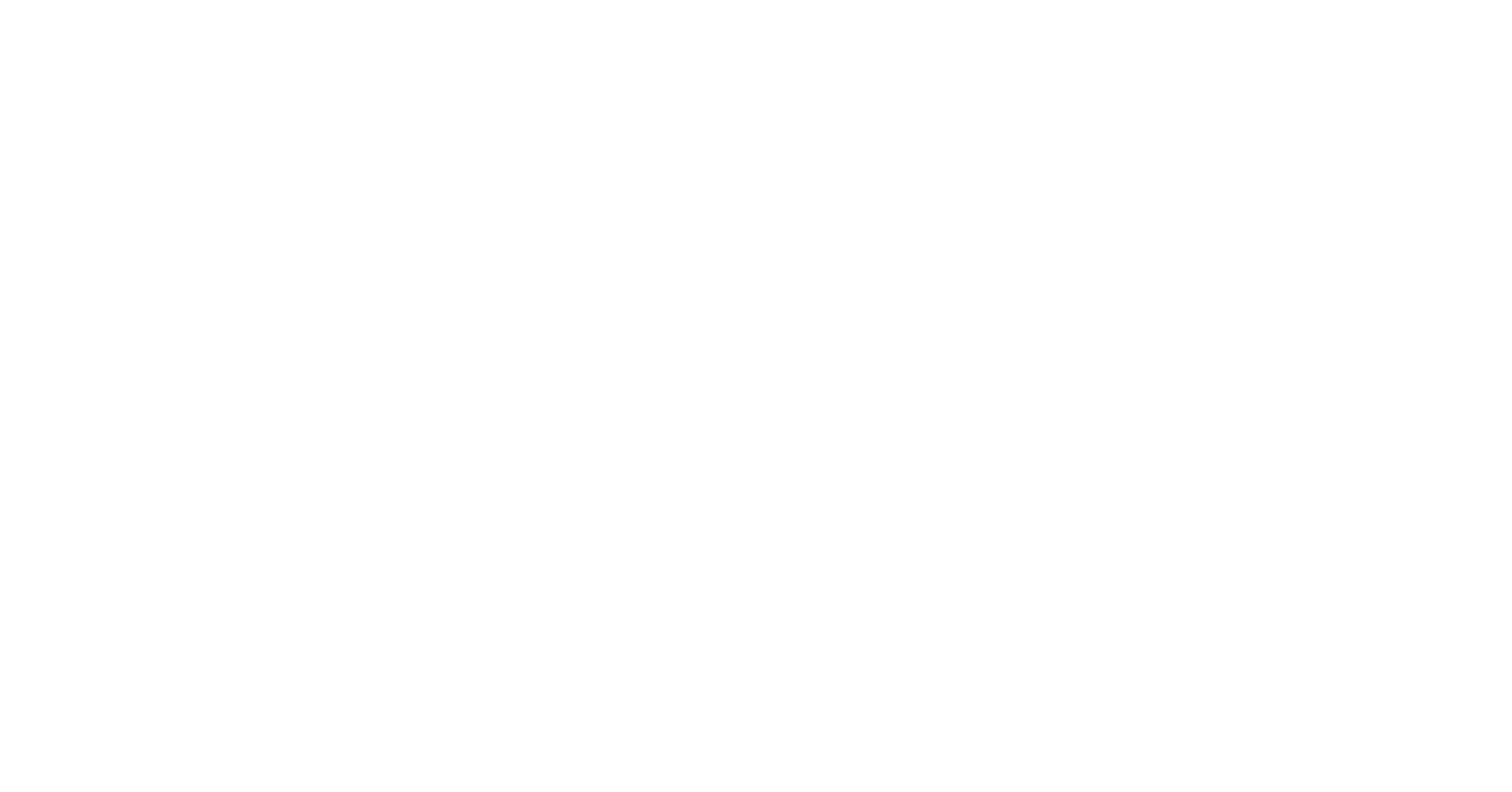
Let .

Differentiating equation (ii) on both sides,

Exercise 39 – 50

Exercise 40

Let .



## 8.6 Numerical Integration

### Trapezoidal Approximation

To approximate , use

where , and is the number of subdivisions.

### Simpsons Approximation

To approximate ,

where and is the number of subdivisions and must be even.

Estimate the minimum number of sub intervals needed to approximate the integral.

If an error of magnitude is less than ,

For Trapezoidal Rule

where is any upper bound of on .

For Simpson’s Rule,

where is any upper bound of on .

Note - is the fourth derivative of .

For Trapezoidal Approximation:

For Simpson’s Approximation:

Exercise 11 – 22

Estimate the minimum number of sub-intervals to approximate the integrals with an error of magnitude less than .

16.

Here,

For trapezoidal approximation, on . (Here, is the maximum value.)

For trapezoidal approximation, we take the next possible integer.

For Simpson’s Rule,

For Simpson’s Rule, we take the next possible even integer.

17.

Here,

For trapezoidal rule, on .

For Simpson’s Rule, on .

Exercise 23, 25

The design of a new airplane requires a gasoline tank of constant cross-sectional area in each wing. The tank must hold of gasoline with a density of . Estimate the length of the tank by Simpson’s Rule.

Horizontal Spacing

Using Simpson’s Rule,

Cross-Sectional Area,

Volume,

Example 2:

Using Simpson’s Rule with , evaluate .

## 8.7 Improper Integrals

Improper Integrals of Type 1: The upper or lower limit is undefined.

, ,

Improper Integrals of Type 2: There is a value within the limits, including the limits, for which the function is undefined.

, ,

To solve such integrals, we must divide the integrals into two parts based on the point where the function becomes undefined, and then integrate for a variable that tends to the value that causes the function to become undefined.

For example,

Example 2:

Example 1:

We know,

(Using L’Hôpital’s Rule)

If the result is a fixed value, it means that the function is convergent. If the result is an undefined value (), the function is divergent.

Example 4:

This is a divergence.

Example 5:

Exercise 1 – 64

Exercise 7

Exercise 23

Exercise 31

Exercise 40

Let .

Exercise 46

Following L’Hopital’s Rule,

Exercise 24

Let .

When , .

When ,